Negative-refraction-like behavior revealed by arrays of dielectric cylinders

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We investigate the electromagnetic propagation in two-dimensional photonic crystals, formed by parallel dielectric cylinders embedded in a uniform medium. The transmission of electromagnetic waves through prism structures is calculated by the standard multiple scattering theory. The results demonstrate that, in certain frequency regimes and when the propagation inside the scattering media is not considered, the transmission behavior mimics the negative refraction expected for a left-handed material. This feature may illusively lead to the conclusion that a negative refraction is observed and it obeys Snell's law of negative refraction. Possible implications for current experimental and theoretical studies of negative refraction are also discussed.

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I. INTRODUCTION

The concept of the so-called left-handed material (LHM) or negative refractive index material was first proposed by Veselago many years ago [1]. In the year 2000, Pendry proposed that a lens made of a LHM can overcome the traditional limitations on optical resolution and therefore make "perfect" images [2]. Since then, the search for LHMs and the study of the properties of LHMs have been skyrocketing, signified by the rapid growth of the related literature.

An earlier realization of a LHM consisted of a composite resonator structure of metallic wires and rings operated in the regime of microwaves [3]. The measurements involved refraction of microwaves by a prism. As pointed out in [4], however, while this work is revolutionary, there are a few unaddressed questions. These include problems associated with near field effects due to either rapid dispersion along the propagation direction [5] or a smaller amount of attenuation on the shorter side of the prism [6]. Moreover, the experiment measured only a single angled prism; this is sufficient only if the *a priori* assumption that the material is refractive is valid. These shortcomings allow for alternative interpretations of the reported experimental data, as discussed in Refs. [5,6].

Ever since its inception, theoreticians have questioned the concept of the LHM and the perfect lenses made of LHMs. The challenge can be classified into two categories. First, scientists have questioned whether left-handed materials are genuinely possible. Second, even if they existed, whether LHMs would really make perfect lenses is also debatable [7–11]. The principle behind the first inquiry is that, although peculiar phenomena observed for some artificial materials may be in conflict with our immediate intuition, if they can still be explained in the framework of current knowledge, it is not necessary to resort to negative refraction or LHMs.

Recently, additional sets of experimental measurements have been reported to provide affirmative evidence on LHMs. For instance, Houck *et al.* [4,12] measured two-dimensional profiles of collimated microwave beams transmitted through composite wire and split-ring resonator prisms. The authors used two angled prisms, in an attempt to refute the alternatives posed in the criticisms on earlier measurements, to obtain a rather consistent negative refractive index. The data appeared to obey Snell's law. The experimental setup in Ref. [4] represents a common method of deducing the refractive index of LHMs.

It has also been shown recently that the negative refraction behavior may also be realized by two-dimensional (2D) photonic crystals (PCs) made from arrays of dielectric cylinders [13,14]. For example, in [14], it is shown that negative refraction can be realized by prism structured photonic crystals made of dielectric cylinders. These discoveries may potentially give many more applications of PCs, and would have strong impact on the future development of science and technology. Motivated by these findings, here we would like to explore further the issue of negative refraction revealed by 2D PCs. We will adapt the prism idea from the experiment observation of composite materials [4] and the previous theoretical study [14] to investigate the transmission behavior of electromagnetic (em) waves through prism-shaped 2D photonic crystal structures.

We believe that there is some confusion in understanding negative refraction. We must stress that the concept of negative refraction originated from the search for LHMs. We notice that in the literature some peculiar refraction behavior has been observed and has been mistakenly connected to the negative refraction expected from LHMs [15,16]. Actually, this kind of refraction is simply due to the anisotropic properties of right-handed materials. We believe that it is important to differentiate the negative refraction caused by the medium anisotropy from the negative refraction due to the properties of LHMs [1]. The latter would be revolutionary with respect not only to technology [2] but also to fundamental physics. Failing to recognize the difference has caused confusion [15,16], as discussed in [17]. In this paper, the concept of negative refraction is discussed in the context of the expectations from LHMs.

Based upon our rigorous simulations, we conclude that a prism measurement or simulation may not be able to discern conclusively the negative refraction expected for LHMs. As a matter of fact, we found that some apparent abnormal refraction that has been thought to be negative refraction may

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be well explained in the context of partial-gap effects or the related Bragg scattering, which are common for all kinds of wave propagation in periodic structures. In this paper, we present some key simulation results to show our perspective. Our results will also shed light on experimental and theoretical studies of negative refraction.

II. THE SYSTEM AND METHOD

Here we consider the transmission of electromagnetic (em) waves in photonic crystals. To make it easy to reproduce our results, we will use the photonic crystal structures that have been commonly used in previous simulations, such as those in [13]. The systems are two-dimensional photonic crystals made of arrays of parallel dielectric cylinders placed in a unform medium, which we assume to be air. The solution for the wave scattering and propagation in such systems can be obtained by multiple scattering theory. This theory is exact and was first formulated systematically by Twersky [18], and has since been reformulated and applied successfully to optical, sonic, and water wave problems [19–21]. The results show that an apparent "negative" refraction is indeed possible, in line with the experimental observation [4]. But this negative refraction is not sufficient to prove that the materials are left handed, and it is not related to the negative refraction expected from LHMs, rather it is due to the anisotropic Bragg scattering of the arrays of dielectric cylinders.

The multiple scattering theory (MST) used in our simulation can be summarized as follows. Consider an arbitrary array of dielectric cylinders in a uniform medium. The cylinders are impinged by an optical source. In response to an incident wave from the transmitting source and scattered waves from other scatterers, each scatterer will scatter waves repeatedly and thus the scattered waves can be expressed in terms of a modal series of partial waves. Regarding these scattered waves as the incident waves to other scatterers, a set of coupled equations can be formulated and computed rigorously. The total wave at any spatial point is the summation of the direct wave from the source and the scattered waves from all scatterers. The intensity of the waves is represented by the modulus of the wave field. Details about MST can be found in Ref. [22].

For brevity, we consider only *E*-polarized waves, that is, the electric field is kept parallel to the cylinders. The following parameters are used in the simulation. (1) The dielectric constant of the cylinders is 14, and the cylinders are arranged in air to form a square lattice. (2) The lattice constant is a and the radius of the cylinders is $0.3a$; in the computation, all lengths are scaled by the lattice constant, so all the lengths are dimensionless. (3) A variety of outer shapes of the arrays is considered, including a prism structures and a slab.

III. RESULTS AND DISCUSSION

First we consider the propagation of em waves through a prism structures of arrays of dielectric cylinders, by analogy with those shown in [4]. Two sizes of prisms are considered and illustrated by Fig. 1. We used three types of source in the simulations: (1) plane waves; (2) collimated waves obtained by guiding wave propagation through a window before incidence on the prisms, mimicking most experiments; (3) a line source. The results from these three scenarios are similar. In this article, we show only the results from a line source which is located at some distance beneath the prisms. We note here that when using a plane wave effects from the prism edges play a role and should be removed.

We have plotted the transmitted intensity fields. The results are presented in Fig. 1. Here the incident waves are transmitted vertically from the bottom. The impinging frequency is $0.192\times 2\pi c/a$; the frequency has been scaled to be nondimensional in the same way as in Ref. [13]. We note that at this frequency the wavelength is about five times the lattice constant, nearly the same as that used in the experiment of [4]. The incidence is along the $[\cos 22.5°, \sin 67.5°]$ direction, that is, the incidence makes an equal angle of 22.5° with regard to the [10,11] directions of the square lattice; the reason we choose this direction will become clear from later discussions. Moreover, on purpose, the intensity imaging is plotted for the fields inside and outside the prisms on separate graphs, which are denoted by 1 and 2 respectively. For the two incidence cases, the fields outside the PCs are plotted in Figs. 1(a1) and 1(b1), and the corresponding fields inside the PCs are plotted in Figs. 1(a2) and 1(b2).

Without looking at the fields inside the prisms, purely from Figs. 1(a1) and 1(b1), we are able to calculate the main paths of the transmitted intensities. The geometries of the transmission are indicated in the diagrams. The tilt angles of the prisms are denoted by ϕ, whereas the angles made by the outgoing intensities relative to the normals of the titled interfaces are represented by θ. According to the prescription outlined in Ref. [4], once ϕ and θ are determined, Snell's law is applied to the intensity field path to determine the effective refractive index: $n\sin\phi=\sin\theta$ (this method will be discussed later). If the angle θ is toward the higher side of the prism with reference to the normal, the angle is considered positive. Otherwise, it is regarded as negative. In light of these considerations, apparent "negative refractions," similar to the experimental observations, indeed appear and are indicated by the black arrows in the graphs. After invoking Snell's law, the negative refraction results are deduced. From Figs. 1(a1) and 1(b1), we obtain negative refractive indices as -0.84 ± 0.17 and -0.87 ± 0.21 for the two prisms. The inconsistency between the two values is less than 4%, which is close to that estimated in the experiment [4]. The overall uncertainty in the present simulation (up to 24%) is less than that in the experiment (up to 36%) [4]. Therefore, a consistent negative refractive index may be claimed from the measurements shown in Figs. 1(a1) and 1(b1). This could be regarded as evidence showing that the photonic structures described by Fig. 1 are left-handed materials, at least for the frequency considered. By the same token, we even found that with certain adjustments such as rotating the arrays or varying the filling factor, the index thus obtained can be close to the perfect -1.

Although tempting, there are ambiguities in the above explanation of the transmission in the context of negative refraction that has led to the negative refractive index. This can

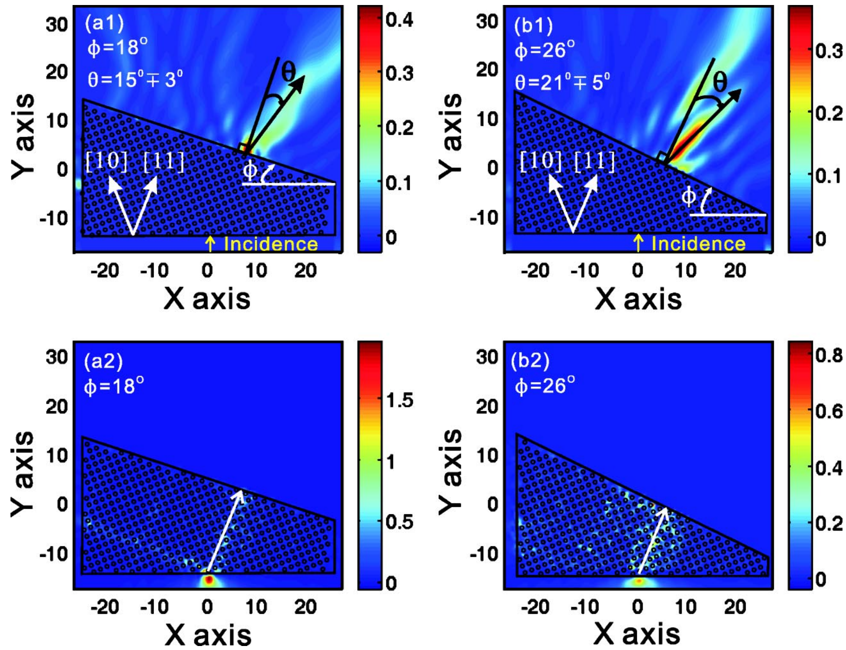

FIG. 1. (Color online) Images of the transmitted intensity fields. Here, the intensities inside and outside the prism structures are plotted separately, so that the images can be clearly shown with proper scales. The geometric measurements can be inferred from the figure. The tilt angles for the two prisms are 18° and 26°, respectively, and have been labeled in the figures. For cases (a1) and (b1), we observe apparent "negative refraction" at the angles of $15\pm3°$ and $21\pm5°$, respectively. When applying Snell's law to the energy intensity path, these numbers give rise to the negative refractive indices of -0.84 ± 0.17 and -0.87 ± 0.21 for (a1) and (b1) separately. In (a2) and (b2), the intensity fields inside the prisms are plotted. Here we clearly see that the transmission has been bent. In the plots, [10,11] denote the axial directions of the square lattice of the cylinder arrays.

be discerned by taking into consideration the wave propagation inside the prisms. As shown clearly by Figs. 1(a2) and 1(b2), the transmission inside the prisms has already been bent at the incidence interfaces, referring to the two white arrowed lines in Figs. 1(a2) and 1(b2). If Snell's law were valid at the outgoing interfaces, it should also be applicable at the incident boundaries. Then with a zero incidence angle and a finite refraction angle, the application of Snell's law to the intensity field, as in the experiment, would lead to the absurd result of an infinite refractive index for the surrounding medium, which is taken as air. In addition, when the bending inside the prism is taken into account, the incident angle at the outgoing or tilted surface is not θ any more. Therefore the index value obtained above cannot be correct. These ambiguities cannot be excluded from the current experimental or theoretical research that has claimed to support LHMs using Snell's law. In the experiments of Ref. [4], for example, the transmission fields inside the prisms are not shown. Another comment may be made with regard to the method of deducing the refractive index. In all previous experimental or theoretical explorations (e.g., Refs. [3,4,12,23]), it was the energy intensity that was applied to Snell's formula. This is valid only when the wave propagation in the medium is isotropic. There is no report showing that the media are isotropic. Instead, some experiments [23] actually show anisotropy. In this case, it is incorrect to use the simple version of Snell's law to obtain the refractive index as in Ref. [23]. For an anisotropic medium, the refractive angle can be deduced by a proper application of Snell's law, which has been detailed in Refs. [17,24].

In search of the cause of the apparent "negative refraction," or the quasi-negative-refraction, shown in Fig. 1, we carried out further simulations. For example, in Fig. 2, we plot the em wave transmission across two slabs of photonic crystals with two different lattice orientations: one is along the diagonal direction, i.e., the [11] direction, and the other is along the direction making an equal angle with the [11,10] directions. The incident frequency is again $0.192\times2\pi c/a$. Here, we see that when the incident wave is along the [11] direction, the transmission follows a straight path inside the slab. For the second case, the propagation direction is bent inside the slab: the major lobe tends to follow the [11] direction, and a minor lobe of transmission appears to make a perpendicular angle with regard to the [11] direction, that is the [−11] direction. As aforementioned, if Snell's law were used to obtain the refractive index, the absurd number of infinity would be deduced.

Figure 2 clearly indicates that there are some favorable directions for waves to travel. This phenomenon is not uncommon for wave propagation in periodically structured media. It is well known that periodic structures can modulate the wave propagation dramatically due to Bragg scattering, a feature fundamental to x-ray imaging. In some situations, the systems reveal complete band gaps, referring to the frequency regime where waves cannot propagate in any direction. On some other occasions, partial band gaps may appear so that waves may be allowed to travel in some directions but not in some other directions. More often, the periodic structures lead to anisotropic dispersions in wave propagation. The whole realm of these phenomena has been well

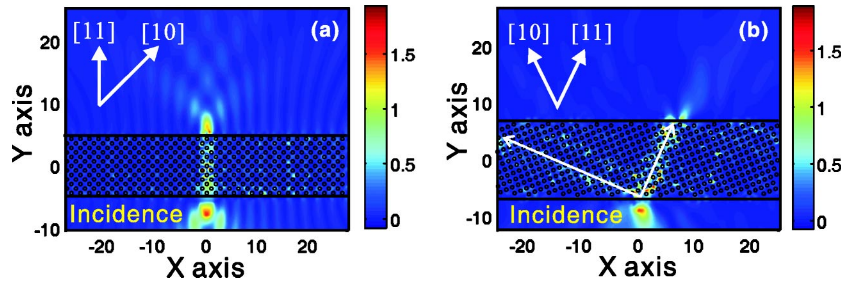

FIG. 2. (Color online) The imaging fields for slabs of photonic crystal structure. Two lattice orientations are considered: (a) The slab measures about 56×10 and the incidence is along the [11] direction; (b) the slab measures 50×13 and the incidence is along the direction that makes an equal angle to the [11,10] directions. The main lobes in the transmitted intensities are shown.

represented by the band structures calculated from Bloch's theorem. The preferable directions for wave propagation are associated with the properties of band structures. In fact, the above abnormal "negative refraction" phenomenon may be explained in the framework of the band structures calculated for the crystal structures considered.

Figure 3 shows the band structure calculated for the square lattices used in the above simulations. A complete band gap is shown between frequencies of 0.22 and 0.28. Just below the complete gap, there is a regime of a partial band gap, in which waves are not allowed to travel along the [10] direction. The frequency $0.192\times 2\pi c/a$ we chose for simulation lies within this partial gap area. Within this gap, waves are prohibited from transmission along the [10] direction. As a result, when incident along an angle that lies between the [11,10] directions, waves will incline to the [11] direction. This observation explains the abnormal refraction observed above. Due to the symmetry, the waves are also allowed to travel in the directions that are perpendicular to [11]. This explains why we also saw an intensity lobe along $[-11]$ in Fig. 2. We found that these results are also qualitatively similar for other frequencies within the partial band gap. Furthermore, we checked other frequencies at which, although there is no partial band gap, the dispersion is highly anisotropic; similar features may also be possible.

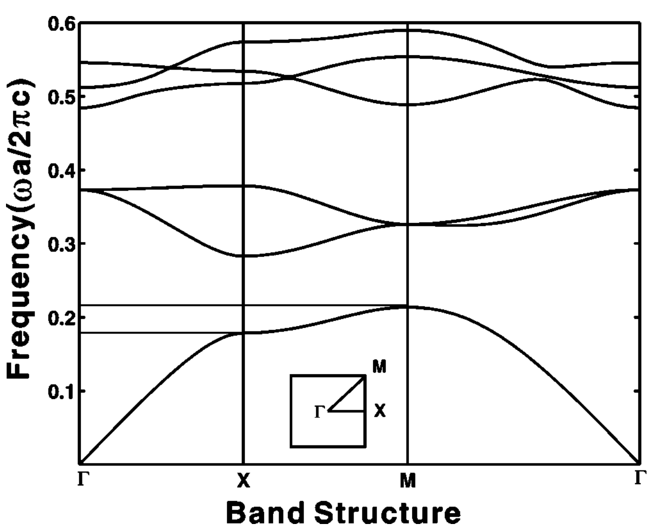

FIG. 3. The band structure of a square lattice of dielectric cylinders. The lattice constant is a and the radius of the cylinders is $0.3a$. ΓM and ΓX denote the [11,10] directions, respectively. There is a partial gap between the two horizontal lines.

The phenomenon of the band structure may help in discussing the abnormal transmission that has been interpreted as the onset of the negative index behavior in the experiments [4]. To date, all the claimed left-handed materials are artificially made periodic composite structures, in which negative permittivity and permeability are obtained near a resonance frequency. From our experience, near such a resonance frequency, either complete band gaps or partial band gaps are likely to appear. In other words, the wave propagation largely depends on the periodic structures. Without taking this fact into account, the transmission may be regarded as unconventional, and then the artificial conclusion that a LHM has been realized and observed may be resorted to. Due to the limited information available from the experiments, we cannot yet definitely come to the conclusion that the apparent "negative refractions" observed in experiments are due to partial band gaps, highly anisotropic dispersions, or inhomogeneities. The present simulations, however, at least indicate that some of these effects must be addressed in order to be able to interpret the experimental data correctly.

Lastly, we would like to make a comment on the permittivity and permeability of composite materials. It is well known that these quantities are meaningful only in the sense of the effective medium theory. A basic assumption is that the wave propagation should not be sensitive to details of the supporting media. Otherwise, whether the definitions of the permittivity and permeability are still meaningful is itself doubtful. If measurements of the two parameters cannot be justified accordingly, it would be misleading to deliberately interpret wave propagation in terms of negative or positive refraction. In the above cases, although the wavelength is much larger than the lattice constant, the wave propagation is obviously still sensitive to the lattice arrangements. Therefore, the flow of energy should be interpreted as mainly controlled by Bragg scattering processes, rather than as propagation in an effective medium. In other words, the medium acts as a device that can bend the flow of optical energies, rather than as an effective refracting medium.

So far, there is no solid experimental evidence to show that composite materials, e.g., from Ref. [3], made of resonant rings and wires are an effective medium. Most of the results are obtained under certain special conditions. These materials are made of periodically arranged macroscaled structures and actually belong to the class of photonic crystals. As indicated by the present results, knowing how the waves propagate within these materials will be essential in discerning whether these materials are actually negatively refracting, or just negatively manipulating em wave transmission. Without knowing the detailed information about the waves inside the medium, ambiguous conclusions can be deduced.

IV. SUMMARY

In summary, we have simulated wave transmission through prism structures. The results have shown some ambiguities in interpreting the apparent abnormal behavior in the transmission as the onset of the negative refraction index behavior. It is suggested that periodic structures can give rise to peculiar phenomena which need not be regarded as negative index behavior. The results help clarify some ambiguities that have been involved in the current debate on negative refraction and LHMs.

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